Measuring Exponential Growth

Skills: Math

Objective: Students reinforce math skills while learning about best practices for protection of water quality in the management of animal feeding operations.

Background

Many of the animal products you buy at the supermarket are from animals raised in animal feeding operations (AFOs). These are places where many animals (usually of the same type) are raised in confined situations.

The people who manage AFOs take steps to manage the wastewater from these facilities so it can benefit crops as fertilizer rather than wash into ponds and lakes. Wastewater from AFOs is filled with nutrients from the manure of the animals, as well as anything else washed down the drain (bedding, spilled feed, etc.).

Nutrients that get into ponds and lakes act as fertilizer to the algae in the ponds, just as nutrients on land act as fertilizer to plant life. When too much wastewater from AFOs enters ponds, lakes, or other surface water, the algae multiplies rapidly, covering the surface of the water. This is called an "algal bloom." When the algae die, bacteria in the water decompose the algae. It takes a great deal of aerobic bacteria to decompose an algal bloom. In the process the oxygen levels in the water decline, killing fish and other aquatic life. This process is called "eutrophication."

To prevent eutrophication, and to keep the manure nutrients for use as fertilizer for crops, AFOs often have large holding lagoons for animal waste. Some AFOs even compost the waste. In cases where many more nutrients are available than the surrounding fields need, some states (such as Oklahoma) have manure transfer programs to move manure to places where it can be used effectively.

Another use for wastewater is as an alternative to petroleum-based natural gas. Anaerobic digesters recover methane from liquid waste which can be used to produce electricity at the same time that it reduces methane emissions.

Activity

1. Read and discuss background and vocabulary.
2. Hand out the worksheet included with this lesson for students to complete.
   —Students make a table to show results of their calculations.
   —Students compare and justify findings with a partner or in a small group. Students will be prepared to explain their reasoning.

P.A.S.S.

GRADE 6
Math Process—1.2; 2.2; 3.3; 4.1; 5.1,4
Math Content—2.4; 5.2

GRADE 7
Math Process—1.2; 2.2; 3.3; 4.1; 5.1,4
Math Content—2.1b,3a; 5.1

GRADE 8
Math Process—1.2; 2.2; 3.3; 4.1; 5.1,4
Math Content—2.1b,2a; 5.1

Resources Needed

calculators

Vocabulary

aerobic—a process or organism that requires oxygen
AFO—animal feeding operation; a place where animals are confined in large numbers and fed (rather than grazing in large pastures)
algal bloom—the quick growth of algae due to excess nutrients in the environment
anaerobic—a process or organism that does not require oxygen
binary fission—the reproduction process for bacteria; asexual
eutrophication—the depletion of oxygen in a pond or lake due to the decomposition of algal blooms by aerobic decomposers; leads to the death of aquatic life
exponential growth—growth of population at increasing rate due to increasing size
fecal—having to do with manure
generation—number of doublings of a bacterial population
lagoon—an enclosure built to hold liquid waste
methane—a gas byproduct of digestion, released by bacteria in the gut

3. Review the different kinds of graphs. (See “Graphs” in the “Resources” section.)
   —Students will select a graph and explain why it would be the best tool for graphing the information from the previous question.

Extra Reading

Eberts, Marjorie, *Nature*, McGraw-Hill, 1996.
Farrell, Jeanette, *Invisible Allies: Microbes That Shape Our Lives,* Farrar, Straus and Giroux, 2005.
Fowler, Allan, *If It Weren’t for Farmers*, Children’s, 1994.
Sayre, April Pulley, *Lake and Pond,* 21st Century, 1997.
Toupin, Laurie, *Freshwater Habitats: Life in Freshwater Ecosystems*, Franklin Watts, 2005.

Name ______________________________________

Measuring Exponential Growth of *E. coli*

One way to tell if there is manure contamination of a water supply is to check for the presence of *Escherichia coli* bacteria, commonly referred to as *E. coli*. This bacteria is found in the intestines of all animals, including humans, and is an indicator of fecal contamination. Bacteria reproduce in a process called binary fission. The bacteria makes a copy of its one chromosome (Humans have 46.), grows longer, and then splits in half. Their population doubles with every generation. This is called exponential growth. Some bacteria, like *E. coli*, can divide every 20 minutes. The number of bacteria present after a certain number of generations is:

$$2^{\text{number of generations}}$$

since one cell divides into two each time. So after 15 generations, one bacteria will have become:

$$2^{15} = 32{,}768 \text{ bacteria}$$

In other words, every five hours, one *E. coli* bacteria becomes 32,768 *E. coli* bacteria! If you want to determine how many bacteria you would end up with if you started out with more than one bacteria, you would just multiply that starting number by the formula above:

$$\text{starting number of cells} \times 2^{\text{number of generations}}$$

If you started with 15 *E. coli* bacteria, after five hours you would have:

$$15 \times 2^{15} = 491{,}520 \text{ bacteria.}$$

1. To see how fast *E. coli* grows, use the information above to calculate the number of bacteria in the first 25 generations.

2. Since *E. coli* divide every 20 minutes until optimal conditions have been reached, how long would it take one *E. coli* to go through 25 generations?

Produced by Oklahoma Ag in the Classroom, a program of the Oklahoma Cooperative Extension Service, the Oklahoma Department of Agriculture, Food and Forestry and the Oklahoma State Department of Education, 2008.

Measuring Exponential Growth of *E. coli* (answers)

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1. To see how fast *E. coli* grows, students use the information above to calculate the number of bacteria in the first 25 generations.

1 = 2	**2 = 4**	**3 = 8**	**4 = 16**
5 = 32	**6 = 64**	**7 = 128**	**8 = 256**
9 = 512	**10 = 1024**	**11 = 2048**	**12 = 4096**
13 = 8192	**14 = 16, 384**	**15 = 32, 768**	**16 = 65, 536**
17 = 131,072	**18 = 262, 144**	**19 = 524,288**	**20 = 1,048,576**
21 = 2,097,152	**22 = 4,194,304**	**23 = 8,388,608**	**24 = 16,777,216**
25 = 33,554,432			

2. Since *E. coli* divide every 20 minutes until optimal conditions have been reached, how long would it take one *E. coli* to go through 25 generations?
   8 hours, 20 minutes

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